Estimating Policy Effects in a Social Network with Independent Set Sampling

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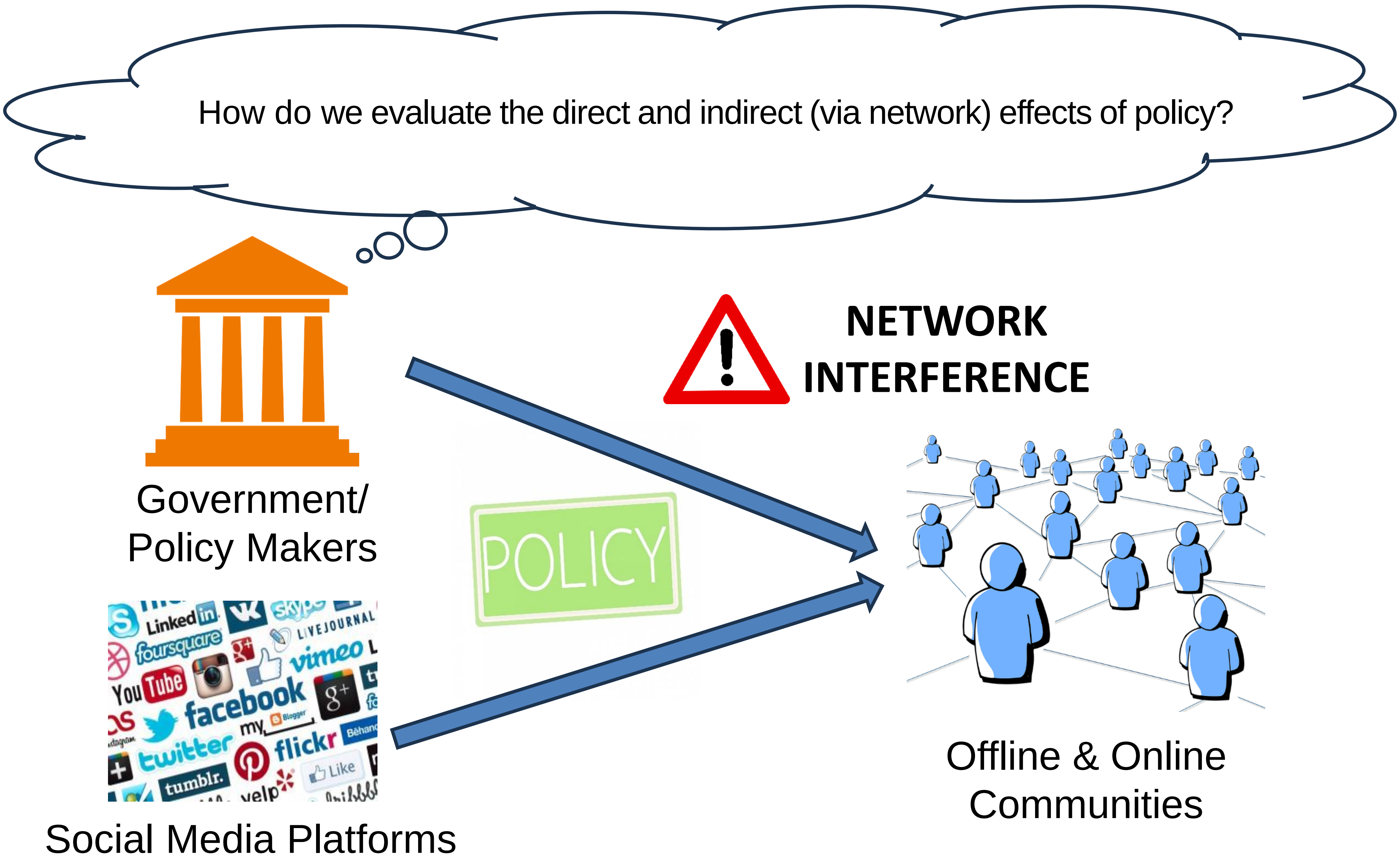

How do we evaluate the direct and indirect (via network) effects of policy?
NETWORK INTERFERENCE
Government/
Policy Makers
POLICY
Social Media Platforms
Offline & Online
Communities

Policy Evaluation Approaches

- Use (quasi) – experimental approaches or conduct RCT on a group of people in the population *(Coly 2017, White 2017)*
    - Affected by network interference and other network-related confounders, such as homophily
- Require appropriate and efficient sampling methods *(Olsen 2013)*
    - Risk of network interference “within” the treatment and non – treatment groups

Network Interference

- Network interference could affect how policies influence the behaviors of the people in the communities, and how their relationships could influence the effect of the policies.

1. Isolate the “direct” effect of the policy change from any “indirect” effect of the policy change via network influence

2. Find the “net” treatment effect of a policy change in the presence of homophily and network influence in the population.

Current Approaches

1. Random Selection with Naïve linear regression *(e.g., Porter 1981)*
    - Regress on observable covariates to explain the policy effects

2. Linear-in-means model *(e.g., Manski 1993, Kline 2012)*
    - Use aggregated values of nodes' neighbors as instrumental variables to explain peer effect

3. Graph clustering selection *(e.g., Ugander 2013)*
    - Sample random clusters in network for external test exposure

4. ERGM / Co – evolution model *(e.g., Wasserman 1996, Snijders 2007)*
    - Model social network structures through specified statistics and properties

Why should we care?

1. Presence of network interference within treatment groups and across groups within network

2. Bias could over / under correct the policy effect
   - Estimation of policy influence is generally confounded with homophily
   - Better manage the resources for the policy implementation

3. Current approaches have certain limitations
   - Econometric approaches: Cannot guarantee the strength of IVs, and might fail for certain network structures
   - Cluster sampling: Vulnerable to network interference “within” sampled clusters

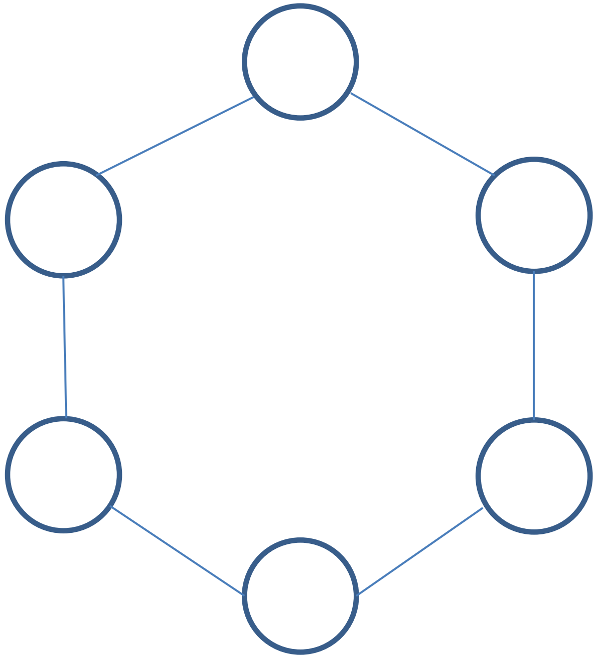
Proposed Methodology

Combines existing work in stochastic actor-oriented models (SAOM) & diffusion contagion models with *independent set sampling technique*

So, what is an independent set?

Independent Set

Definition: A set of vertices S is called an independent set if no two vertices in this set S are adjacent to each other



Let's call this graph G, technically it's C_6.
Let construct some independent set S

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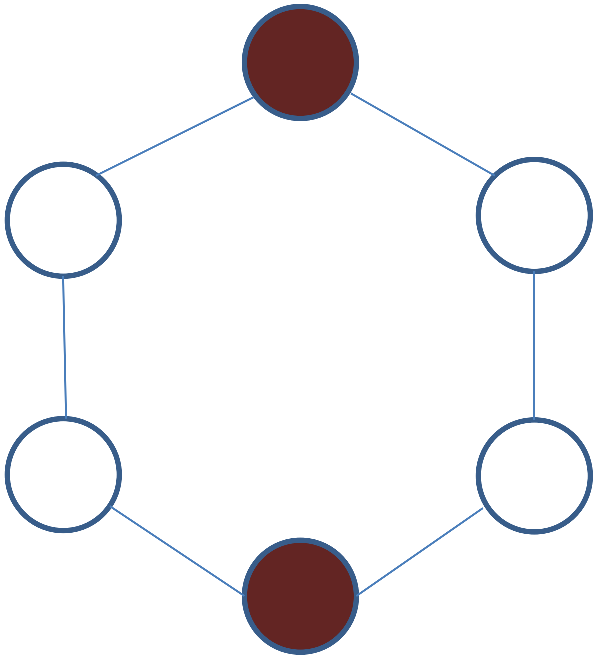
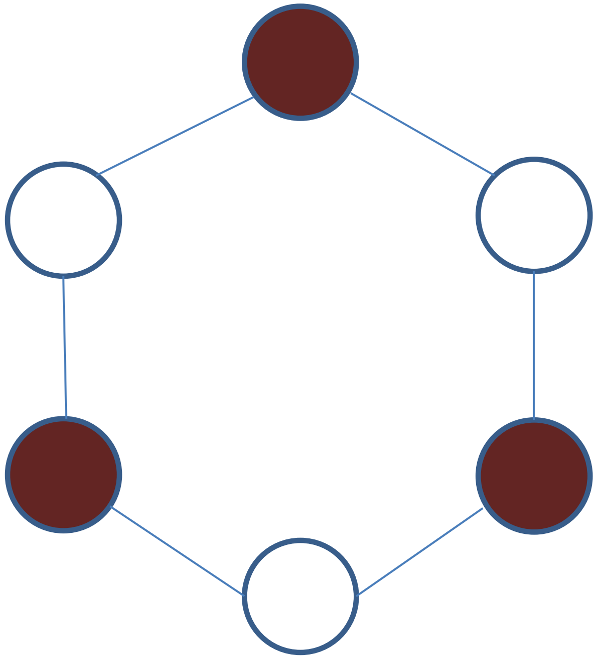


This is one possible S.
It is a **maximal** independent set.



This is another possible S.
It is a **maximum** independent set.

Bounds

Let $G = (V, E)$, $|V| = n, |E| = e, d_v$ be degree of vertex v, Δ is the maximum degree, $\alpha(G)$ is the size of the maximum independent size

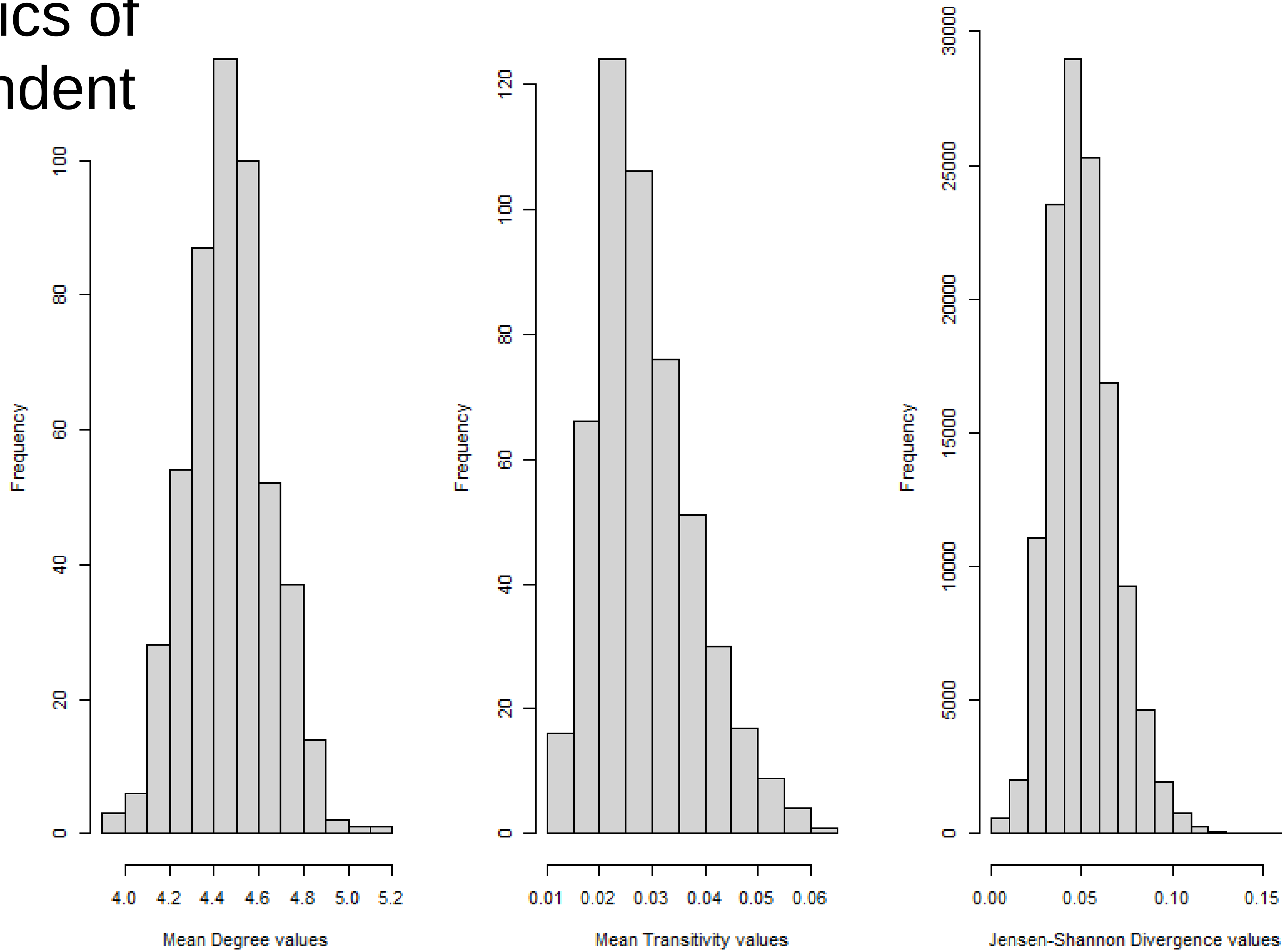
Theorem (Kwok): $\boldsymbol{\alpha(G) \leq n - \frac{e}{\Delta}}$

Theorem (Caro – Wei): $\boldsymbol{\alpha(G) \geq \sum_{v \in V} \frac{1}{1+d_v}}$

Guarantee on the size of the largest independent set sample

- Flexible to choose a suitable sample size given their resource constraints and objective

Structural Statistics of Maximal Independent Sets



Mean Degree: 4.466
Mean Transitivity: 0.028
Jenson-Shannon Divergence: 0.049

Why use independent set sampling?

- Social networks are known to be sparse and have bounded degree
  - Independent sets can be used to sample large numbers of nodes relatively efficiently

- Obtain a more representative sample of a network
  - Ensure that the sample is not overly influenced by the presence of dense subgraphs in the original network

- Eliminate interactions within sample groups and isolate policy effects
  - Through such construction, it avoids selecting connected groups of nodes, so it reduces the chances of treatment spill-over in any such sample from the network

- Better identification of network formation within sample (due to policy change)
  - Since the sample is isolated by construction, any network formation can be attributed to homophily of being exposed to treatment (or due to chance)

So, the plan is…

1. For a given network, we find an independent set, cluster sample and random sample to be exposed to the treatment

2. Simulate the joint evolution of the network and behavior using a stochastic co-evolution model

3. Obtain estimates for homophily and influence; compare across the 3 samples

4. On top of it, we use a second – order difference approach to measure policy treatment effect

Simulation Study

➢ Want to investigate the effects on focal behavior due to the policy

- Use a popular dataset accessed from the *wooldridge* package in R *(with random assignment on gender)*

- Model changes in focal behavior using a logistic regression based on individual covariates (policy – increase price level of goods)

- Create 3 waves (4 stages) to simulate evolution
  1. Initialize random scale free network
  2. Choose an independent set sample/random sample/cluster sample with small noise
  3. Change behavior according to logistic regression (no change in network)
  4. Parameterize the evolution based on certain probabilities of change

SAOM

- Jointly model the co – evolution of networks and behaviors using a CTMC
- At every micro-step, there is at most one change in the respondents' focal behavior or the edges in the network
- Model the opportunities to form/delete tie or focal behavior to follow a Poisson process with different rate functions
- Individuals optimize own's objective function to determine specific changes in network or behavior
    - Evaluation function: Measure each respondent's utility
    - Endowment function: Capture loss in utility – loss in ties or behavior which is gained earlier
    - Noise function: Represent a portion of respondent's preference

SAOM for our study

- Assume loss in utility is equal to respondent’s earlier gain. No endowment function

- Model rate and objective functions with network statistics such as
  1. Degree
  2. Transitivity
  3. Homophily based on the respondent’s covariates (focal behavior and price)

  and behavioral statistics such as
  1. Similarity measure
  2. Behavioral tendency effect
  3. Peer influence effect

  Note: these are not exhaustive, and exact selection will depend on the specific problem context

Stronger homophilous effect

Estimates, standard errors				
Network Dynamics		Independent	Random	Cluster
1	Friendship rate (Period 1)	0.0067** (0.0047)	0.0065** (0.0047)	0.0064** (0.0046)
2	Friendship rate (Period 2)	0.1000 (NA)	0.1000 (NA)	0.1000 (NA)
3	Friendship rate (Period 3)	0.0302*** (0.0099)	0.0101* (0.0057)	0.0234** (0.0059)
4	Transitivity	-1.9489 (1.0932)	0.4378 (0.5195)	-0.2196 (0.5550)
5	**Behavior homophily**	**2.2641* (1.3203)**	**1.1380 (1.3819)**	**2.0268 (1.3823)**
6	**Policy exposure homophily**	**3.4221* (2.0459)**	**1.6874 (2.6559)**	**1.8289 (1.9251)**
Behavior Dynamics				
7	Behavior rate (Period 1)	0.1000 (NA)	0.1000 (NA)	0.1000 (NA)
8	Behavior rate (Period 2)	0.9382*** (0.1326)	0.7567*** (0.0975)	0.6685*** (0.1015)
9	Behavior rate (Period 3)	0.0651 (0.0384)	0.1000 (NA)	0.0344 (0.0244)
10	Behavior Tendency (Linear Shape)	-3.7529*** (0.7474)	-3.4438*** (1.0649)	-3.8396 (4.1641)
11	**Average Peer Influence**	**0.1394 (1.5125)**	**-1.9052 (1.8527)**	**2.4923 (5.3836)**
12	Outdegree	0.0693 (0.0612)	0.0439 (0.0296)	0.0488 (0.1597)

- A higher price level serves as a proxy for being included in the treatment set
- Observe a higher homophily based on focal behaviour and price level for independent set

Estimating treatment effect

- Compute differences in the proportion of individuals having the focal behavior in both the treatment and non-treatment groups

- Track the difference estimates over 4 time periods

  (A) Before the policy implementation

  (B) Right after the policy implementation

  (C) After one wave of simulated evolution

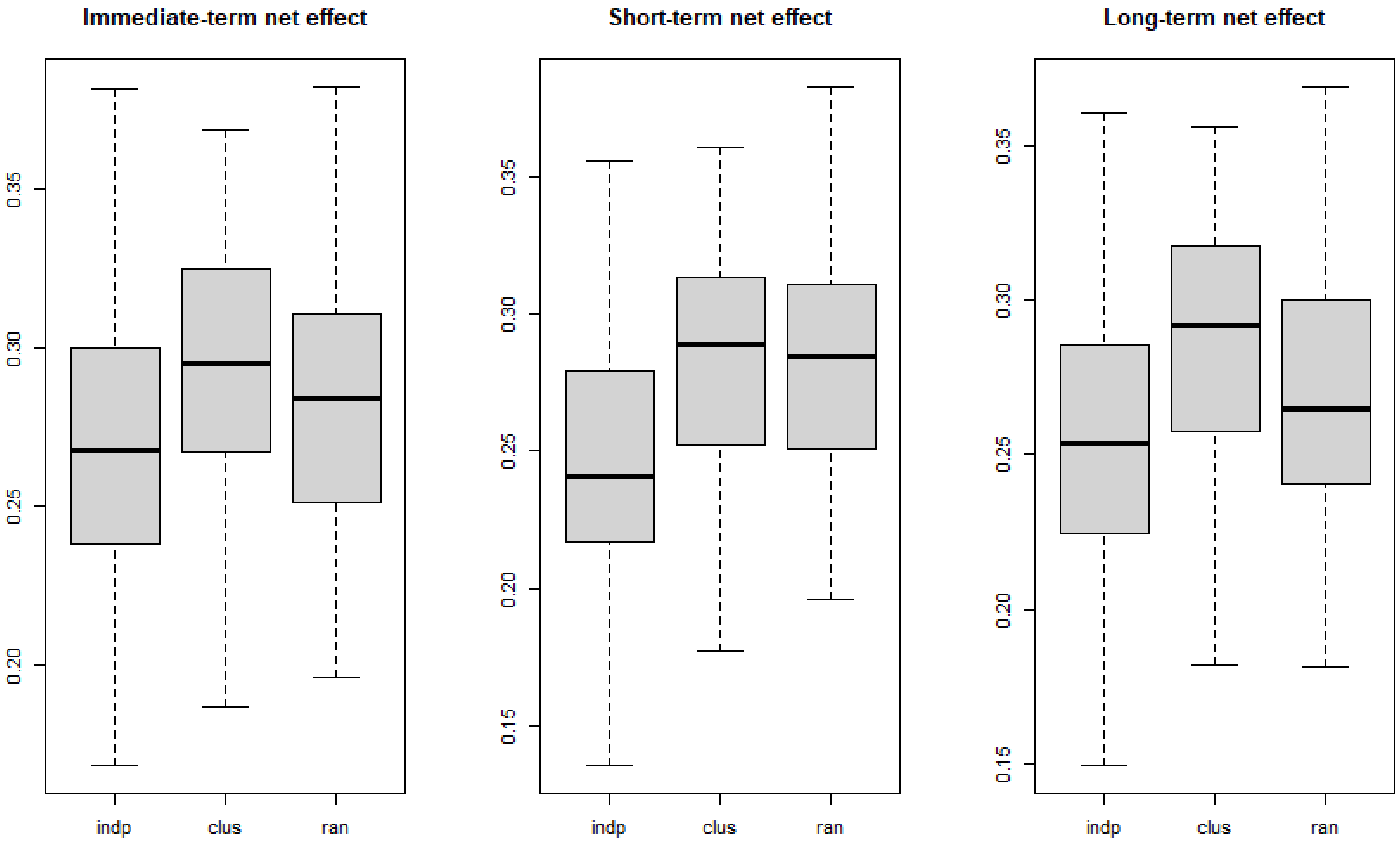
  (D) Future epochs of the predicted networks based on SAOM

- Immediate term net treatment effect: B-A

  Short-term net treatment effect: C-A

  Long-term net treatment effect: D-A

Direct and *net* treatment effect of policy



Key findings

1. Through independent set sampling, we eliminate any network interference within the treatment group
   - Decouple direct and indirect policy effects in the immediate term
   - Leads to better estimation of the treatment effect
   - Smaller policy treatment effect in the immediate and short term

2. Tendency for respondents to form ties with others who have similar focal behavior, especially with those in the same treatment group

3. Policy makers can spend less resources by exposing the policy on an independent sample and let the network do the work

Future Work

1. What if we obtain a weakly independent set sample due to incomplete data/unobservable links?

2. Do size/certain centrality measures in the independent set affect the speed of influence/coverage?

3. Since the construction of independent set is affected by the graph structure, how would different graph structure affect the effectiveness of such sampling?

4. Which is the “best” independent set to use, in terms of cost of policy implementation or rate of coverage?

Thank you. Any questions?

https://arxiv.org/abs/2306.14142

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